8/27/25 Lecture 2

Linear System → a list of equations

$$\left.\begin{aligned} 2x_1 - 3x_2 \quad\quad &= 1 \\ x_1 + x_2 + 4x_3 &= 5 \end{aligned}\right\} \text{2x3 linear system}$$

a_{12} = coefficient

row column

<u>Goal</u> - find all $x_1, \dots, x_n$ that satisfy all eqations

$\boxed{\text{Ex}}$

$$\begin{aligned} x_2 + 4x_3 &= -5 \\ x_1 + 3x_2 + 5x_3 &= -2 \\ 3x_1 + 7x_2 + 9x_3 &= 6 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 0 & 1 & 4 & -5 \\ 1 & 3 & 5 & -2 \\ 3 & 7 & 9 & 6 \end{array}\right] \xrightarrow{??} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \star \\ 0 & 1 & 0 & \star \\ 0 & 0 & 1 & \star \end{array}\right]$$

* Swap rows 1 & 2

$$\left[\begin{array}{ccc|c} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 3 & 7 & 9 & 6 \end{array}\right]$$

* Add $(-3)R_1$ to third row

$$\left[\begin{array}{ccc|c} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 0 & -2 & -6 & 12 \end{array}\right]$$

* Add $(2)R_2$ to third row

$$\left[\begin{array}{ccc|c} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 0 & 0 & 2 & 2 \end{array}\right]$$

"echelon form"
(1^{st} non zero entry in each row is to the left of 1^{st} non zero entry in next row)

Pivot → 1^{st} non-zero entry

* Multiply $(\frac{1}{2})R_3$

$$\left[\begin{array}{ccc|c} 1 & 3 & 5 & -2 \\ 0 & 1 & 4 & -5 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

* Add $(-4)R_3$ to the 2nd row and add $(-5)R_3$ to the first row

$$\left[\begin{array}{ccc|c} 1 & 3 & 0 & -7 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

* Add $(-3)R_2$ to the first row

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 20 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\boxed{\text{ANS} \quad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 20 \\ -9 \\ 1 \end{pmatrix}}$$

In particular $\exists$ 1 solution

"$\exists$" → there exists

Ex 2

$$\left[\begin{array}{ccc|c} 0 & 1 & 4 & -5 \\ 1 & 0 & 1 & 3 \\ 2 & 2 & 10 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 4 & -5 \\ 2 & 2 & 10 & 0 \end{array}\right] -2(R_1)-2(R_2)$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 1 & 3 \\ 0 & 1 & 4 & -5 \\ 0 & 0 & 0 & 4 \end{array}\right]$$

"inconsistent system" because of 3rd row

Ex 3

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 & -1 \\ 1 & 1 & 1 & 2 & 1 \end{array}\right] + (-1)R_1$$

$$\rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 2 & -2 \end{array}\right] + (-1)R_2$$

$$\rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] + (-1)R_3$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$$

This is as far as you can go, and it is called <u>reduced row echelon form</u>.

- ↳ echelon form
- ↳ all pivots = 1
- ↳ all other entries in a pivot column are 0

This means:

$$\left.\begin{aligned} x_1 + x_2 &= 3 \\ x_3 &= 0 \\ x_4 &= -1 \end{aligned}\right\}$$

Soln:

$$\boxed{\begin{aligned} x_1 &= 3 - x_2 \\ x_2 &= x_2 \text{ (free variable)} \\ x_3 &= 0 \\ x_4 &= -1 \end{aligned}}$$